

Kähler potentials of chiral matter fields for Calabi-Yau string compactifications

Joseph P. Conlon, Daniel Cremades and Fernando Quevedo

*DAMTP, Centre for Mathematical Sciences,
Wilberforce Road, Cambridge, CB3 0WA, U.K.
E-mail: j.p.conlon@damp.cam.ac.uk, d.cremades@damp.cam.ac.uk,
f.quevedo@damp.cam.ac.uk*

ABSTRACT: The Kähler potential is the least understood part of effective $N = 1$ supersymmetric theories derived from string compactifications. Even at tree-level, the Kähler potential for the physical matter fields, as a function of the moduli fields, is unknown for generic Calabi-Yau compactifications and has only been computed for simple toroidal orientifolds. In this paper we describe how the modular dependence of matter metrics may be extracted in a perturbative expansion in the Kähler moduli. Scaling arguments, locality and knowledge of the structure of the physical Yukawa couplings are sufficient to find the relevant Kähler potential. Using these techniques we compute the ‘modular weights’ for bifundamental matter on wrapped D7 branes for large-volume IIB Calabi-Yau flux compactifications. We also apply our techniques to the case of toroidal compactifications, obtaining results consistent with those present in the literature. Our techniques do not provide the complex structure moduli dependence of the Kähler potential, but are sufficient to extract relevant information about the canonically normalised matter fields and the soft supersymmetry breaking terms in gravity mediated scenarios.

KEYWORDS: Superstrings and Heterotic Strings, Flux compactifications, Supersymmetry Breaking, Superstring Vacua.

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1. Introduction

Extracting the form of the effective four-dimensional field theory corresponding to compactifications of string theory has been one of the most active areas of research in string phenomenology over the years [1–5]. For $N = 1$ supersymmetric compactifications¹ we know that the effective supergravity theory depends on the Kähler potential $K(\Psi, \Psi^\dagger)$, the superpotential $W(\Psi)$ and the gauge kinetic function $f(\Psi)$, where Ψ represents the chiral superfields surviving at low energies. These include both the charged matter superfields C and the singlet moduli superfields Φ . It is well known that W and f , being holomorphic, are under much better control than the real function K . In particular K is not protected by the standard non-renormalization theorems of $N = 1$ supersymmetry.

The standard way to extract the functional form of K , W and f at tree-level is by dimensionally reducing the original 10D theory, having carefully identified the appropriate 4D superfields in terms of the corresponding 10D geometrical quantities (see for instance [5]). This allows the determination of K , W and f as functions of the moduli fields and some of the matter fields. However there are some matter fields for which dimensional reduction cannot provide the Kähler potential. These include the twisted sector fields of orbifold and

¹Although the validity of our discussion is general, we will concentrate mostly on Calabi-Yau orientifold compactifications of type IIB string theory.

orientifold compactifications, bifundamental fields from magnetised D7 branes and those stretching between D3 and D7 branes. For these cases explicit string amplitudes need to be computed in order to extract the correct Kähler potential. These calculations are essentially limited to flat backgrounds such as toroidal orientifolds [6–8], severely limiting the information that can be extracted. In particular an explicit calculation in a IIB Calabi-Yau orientifold seems out of reach.

The importance of knowing the Kähler potential for the physical matter fields is clear: it is needed for correctly identifying the canonically normalised fields and therefore determines the structure of most of the observable physical quantities, such as the corresponding scalar potential, the Yukawa couplings, etc. In particular, the matter Kähler potential plays a crucial role in the determination of soft supersymmetry breaking terms.

On this regard let us be more specific. There has been much recent effort in understanding supersymmetry breaking in string compactifications. This follows on the progress made in moduli stabilisation [9, 10], which allows the moduli potential to be computed from first principles. The moduli breaking supersymmetry can be identified explicitly and their F-terms evaluated. Prior to this and in the absence of explicit moduli potentials,² it was necessary to parametrise supersymmetry breaking as S -, T - or U - dominated, where S is the 4D dilaton, T the Kähler moduli and U the complex structure moduli. Scenarios of supersymmetry breaking were then constructed and analysed without an explicit moduli potential [11]. In that the discussion of supersymmetry breaking now involves explicit moduli potentials, much technical progress has been made.

However one major obstacle in phenomenological analyses has remained, which is the lack of knowledge of the Kähler metric for Standard Model matter fields. The computation of soft terms starts by expanding the superpotential, metric and gauge kinetic functions as a power series in the matter fields,

$$W = \hat{W}(\Phi) + \mu(\Phi)H_1H_2 + \frac{1}{6}Y_{\alpha\beta\gamma}(\Phi)C^\alpha C^\beta C^\gamma + \dots, \tag{1.1}$$

$$K = \hat{K}(\Phi, \bar{\Phi}) + \tilde{K}_{\alpha\bar{\beta}}(\Phi, \bar{\Phi})\bar{C}^\alpha C^\beta + [Z(\Phi, \bar{\Phi})H_1H_2 + h.c.] + \dots, \tag{1.2}$$

$$f_a = f_a(\Phi). \tag{1.3}$$

C^α denotes a matter field and Φ a modulus field. In the explicit expressions for soft terms, the matter metric $\tilde{K}_{\alpha\bar{\beta}}$ plays a crucial role. This quantity is non-holomorphic, and thus unprotected and hard to compute. However it plays a central role as it determines both the normalisation of the matter fields and their mass basis. In general, an arbitrary form of $\tilde{K}_{\alpha\bar{\beta}}$ can lead to large flavour-changing neutral currents and off-diagonal A-terms. In the absence of other information, $\tilde{K}_{\alpha\bar{\beta}}$ is often assumed to be diagonal and moduli-independent. However, this assumption clearly does not hold for string compactifications where $\tilde{K}_{\alpha\bar{\beta}}$ is a complicated function of the moduli. Given its importance for phenomenological applications, obtaining control over the functional form of $\tilde{K}_{\alpha\bar{\beta}}$ is one of the most important problems in string phenomenology.

²Moduli potentials were actually studied in the past but without fixing all moduli and with no explicit control on hierarchies.

As mentioned before, explicit string CFT calculations such as [12] have been used in toroidal compactifications to work out the matter metrics for adjoint, Wilson line and bifundamental matter. For Calabi-Yau cases, dimensional reduction of D-brane actions has allowed the determination of Wilson line and adjoint scalar metrics [5]. But so far there exist very few results for Kähler metrics for bifundamental matter on Calabi-Yau backgrounds. These are probably the most important phenomenologically since these are the standard chiral fields in D-brane models which will include the quarks and leptons as well as their superpartners.

The purpose of this paper is to give new techniques, applicable to Calabi-Yau backgrounds, for computing Kähler matter metrics. The approach is to compute the modular dependence of $\tilde{K}_{\alpha\bar{\beta}}$ by studying the modular dependence of the physical Yukawa couplings. In certain cases this can be determined easily through dimensional reduction. However in supergravity this is related to the matter metrics, and it is this that will allow us to determine the modular weights³ of $\tilde{K}_{\alpha\bar{\beta}}$. Our main application will be to use these techniques to compute modular weights for bifundamental matter on wrapped magnetised D7 branes in the Calabi-Yau geometries of the large-volume models of [14, 15].

This paper is structured as follows. In section 2 we outline the philosophy of our approach. We describe the calculational approach and the conditions on a modulus for its modular weight to be determined using the techniques of this paper. We also present a one-dimensional toy example to illustrate the techniques and show its relation to the IIB compactifications that are our main interest. In section 3 we apply our approach first to the large-volume models of [14]. We determine the modular weight of the overall volume and describe how the modular weight of the small cycles can also be computed under certain assumptions of the brane geometry. We then apply the same arguments to the toroidal case. The results we obtain are consistent with the explicit computations of [12]. In section 6 we conclude.

2. Philosophy of the approach

To simplify the notation we will consider diagonal matter metrics, although the argument and results holds for the general case. This assumption simplifies (1.3) to

$$K = \hat{K}(\Phi, \bar{\Phi}) + \sum_{\alpha} \tilde{K}_{\alpha}(\Phi, \bar{\Phi}) C^{\alpha} \bar{C}^{\bar{\alpha}} + [Z(\Phi, \bar{\Phi}) H_1 H_2 + h.c.] + \dots \quad (2.1)$$

Using (2.1) we can define the canonically normalised matter fields \hat{C}^{α} . These are related to the unnormalised fields C^{α} by

$$\hat{C}^{\alpha} = \tilde{K}_{\alpha}^{\frac{1}{2}}(\Phi, \bar{\Phi}) C^{\alpha}. \quad (2.2)$$

³We follow standard conventions in calling the powers of moduli fields in the Kähler potential the modular weights of the corresponding matter fields. The name came from the transformation properties of the corresponding field under (toroidal) T -duality [13] in heterotic models.

The approach we take stems from the supergravity formula for the physical (i.e. normalised) Yukawa couplings,

$$\hat{Y}_{\alpha\beta\gamma} = e^{K/2} \frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{\frac{1}{2}}}. \tag{2.3}$$

(2.3) implies that information about the modular dependence of the matter metrics is encoded in the modular dependence of the physical Yukawas \hat{Y} , which may be relatively easy to compute directly. Our aim is to use (2.3) in order to compute the modular dependence of \tilde{K}_α .

This approach could yield no useful information if the modular dependence of the superpotential Yukawas $Y_{\alpha\beta\gamma}$ were unknown. If this were the case, the problem would be overdetermined. We would be unable to separate the functional dependence of the superpotential Yukawa couplings $Y_{\alpha\beta\gamma}$ and the metric dependence $\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma$. Even knowing the full functional form of the physical Yukawa couplings would give no definite information about the functional form of the matter metrics. However in many cases this dependence is known. Certain moduli are forbidden from appearing in $Y_{\alpha\beta\gamma}$, and in this case the scaling behaviour of $\hat{Y}_{\alpha\beta\gamma}$ can be directly related to that of \tilde{K}_α .

Our particular interest here is in IIB flux compactifications. In this case the Kähler moduli T_i are forbidden from appearing in the tree-level superpotential. This can be understood from the Peccei-Quinn shift symmetry

$$\text{Im}(T_i) \rightarrow \text{Im}(T_i) + \epsilon_i, \tag{2.4}$$

which is unbroken perturbatively. As the superpotential is holomorphic, a perturbative dependence on $\text{Re}(T)$ will also give a perturbative dependence on $\text{Im}(T)$, violating the shift symmetry. The non-renormalisation theorems then imply that the T_i do not appear in the superpotential - and thus the Yukawa couplings $Y_{\alpha\beta\gamma}$ - to any order in perturbation theory. It is this that makes it feasible to compute the modular weights of \tilde{K}_α with respect to the Kähler moduli. The complex structure moduli do however enter the tree-level superpotential, and so it is not possible to extract any information about $\tilde{K}_\alpha(U_a)$ from $\hat{Y}_{\alpha\beta\gamma}(U_a)$ (even supposing this could be calculated). The techniques of this paper will apply only to those moduli (such as T_i) that are forbidden from appearing in $Y_{\alpha\beta\gamma}$ and not to the moduli (such as U_a) that do appear in $Y_{\alpha\beta\gamma}$.

Moduli (such as the T-moduli) that do not appear perturbatively in the superpotential may appear nonperturbatively [10, 16]. This may then give nonperturbative corrections to the Yukawa couplings. Such corrections will not affect the results of this paper. This is because our analysis is carried out in a limit of large cycle volume, in which nonperturbative corrections to the Yukawas are exponentially suppressed compared to the tree-level Yukawas. In IIB the tree-level Yukawas are given by a classical wavefunction overlap, which will be dominant at large volume. Our analysis will determine the classical modular weights; small nonperturbative corrections to the Yukawas will give small nonperturbative corrections to the modular weights. In cases where the Yukawas arise solely from nonperturbative effects - as in IIA compactifications - the techniques described in this paper naively do not apply, as there are no classical Yukawa couplings.

Our main interest is in strings supported on magnetised D7 branes. D7 branes wrap 4-cycles whose size is given by $\text{Re}(T_i)$. In the dilute flux approximation, the gauge coupling is given by the size of the cycle. The statement that the gauge theory is weakly coupled is equivalent to the statement that the cycle size is large. In this case the matter metric can be expanded as a power series in $\tau_i = \text{Re}(T_i)$,

$$\tilde{K}_\alpha = \tau_i^\lambda \tilde{K}_0(U_a) + \tau_i^{\lambda-1} \tilde{K}_1(U_a) + \dots \tag{2.5}$$

τ_i contains a factor $e^{-\phi} = g_s^{-1}$ and thus the higher terms in (2.5) can be interpreted as loop corrections. Through (2.3) the modular weight λ determines the scaling of the physical Yukawa couplings with the cycle volume. Thus the computation of λ reduces to the computation of the scaling of the physical Yukawa couplings with cycle volume.

The techniques we will use below are:

1. In a large volume compactification one of the Kähler moduli is much larger than the other ones and determines the overall volume. We can then concentrate on the leading power of inverse volume in the Kähler potential. Matter fields are localised on one of the smaller cycles and so we can use locality to restrict the dependence of the Kähler potential, as rescaling the volume should not change the physical Yukawa couplings. As we know the relation between Yukawa couplings and Kähler potentials, this provides information about the volume dependence of the Kähler potentials.
2. Our fundamental calculational tool is the viewpoint that physical Yukawa couplings arise from the triple overlap of normalised wavefunctions. Due to the constraints of supersymmetry and holomorphy, these wavefunctions can only depend in a simple fashion on the Kähler moduli: classically these enter the normalisation only as an overall scale as in (2.5). The detailed form of the wavefunctions, giving rise to flavour and textures, come from the complex structure moduli, which enter into the superpotential $Y_{\alpha\beta\gamma}(U_a)$. The point is that the overlap integral has a simple dependence on the Kähler moduli and its scaling can be computed without having to compute $Y_{\alpha\beta\gamma}(U_a)$.

This understanding of Yukawa couplings as due to the triple overlap of normalised wavefunctions is both intuitive and supported by explicit calculation. In section 3.1 we shall describe below how it arises directly in the dimensional reduction of higher dimensional Yang-Mills theories - these are the low-energy limits of magnetised brane constructions.

As a warm-up, we illustrate the above approach with a one-dimensional toy example, pointing out the correspondences between it and the more realistic IIB Calabi-Yau flux compactifications subsequently considered.

2.1 A one-dimensional toy model

The toy model consists of particle states on a 1-dimensional line $x = -\infty \rightarrow \infty$. Particles are assumed to be localised about defects on the line located at ξ_a , ξ_b and ξ_c . The wavefunctions are assumed to be Gaussian and of equal width a . We take an infinite line

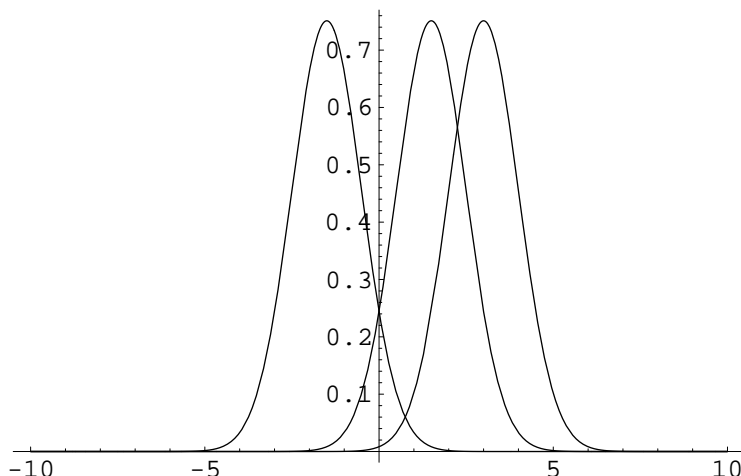


Figure 1: A one-modulus toy model, illustrating three Gaussian matter wavefunctions. The Yukawa coupling is determined by the integrated overlap of the normalised wavefunctions.

for convenience, but because of the rapid wavefunction falloff we may imagine identifying the points $x = -100$ (say) and $x = 100$ without affecting the physics. The normalised wavefunctions are

$$\psi_a(x) = \frac{1}{\pi^{1/4} a^{1/2}} e^{-\frac{(x-\xi_a)^2}{2a^2}}, \tag{2.6}$$

$$\psi_b(x) = \frac{1}{\pi^{1/4} a^{1/2}} e^{-\frac{(x-\xi_b)^2}{2a^2}}, \tag{2.7}$$

$$\psi_c(x) = \frac{1}{\pi^{1/4} a^{1/2}} e^{-\frac{(x-\xi_c)^2}{2a^2}}. \tag{2.8}$$

The forms of these wavefunctions are illustrated in figure 1 for $\xi_a = 1.5, \xi_b = 3, \xi_c = -1.5$ and $a = 1$. The Yukawa coupling is given by the triple overlap of the normalised wavefunctions,

$$\begin{aligned} \hat{Y}_{abc} &= \int_{x=-\infty}^{\infty} dx \psi_a(x) \psi_b(x) \psi_c(x) \\ &= \int_{x=-\infty}^{\infty} dx \frac{1}{\pi^{3/4} a^{3/2}} e^{-\frac{-(x-\xi_a)^2 - (x-\xi_b)^2 - (x-\xi_c)^2}{2a^2}}. \end{aligned} \tag{2.9}$$

The Yukawa coupling is exponentially sensitive to the values of the ξ_i . In the correspondence with IIB compactifications, ξ_i/a corresponds to the complex structure moduli, determining the shape of the wavefunctions, whereas the size of the line corresponds to the Kähler moduli.

We now consider rescaling the size of the line (the ‘Kähler modulus’), without altering the shape of the wavefunctions (the ‘complex structure moduli’). This corresponds to scaling $x \rightarrow \lambda x$. In order for the relative positions and shapes of the wavefunctions to be unaltered, we must also rescale $\xi_a \rightarrow \lambda \xi_a$ and $a \rightarrow \lambda a$. This leaves the relative breadth and

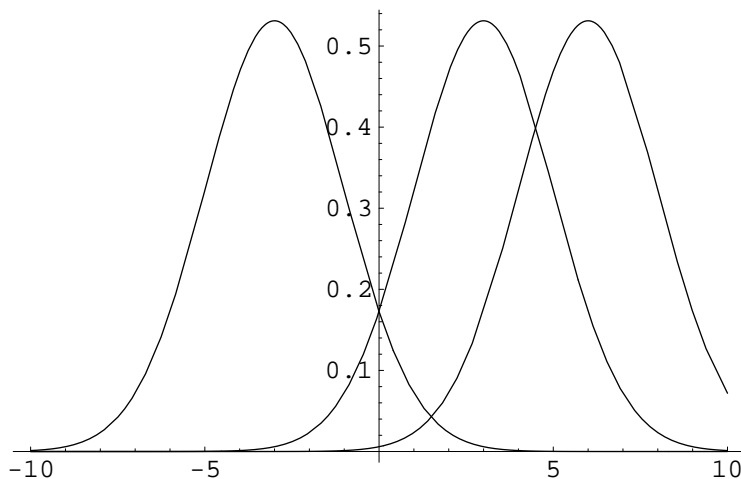


Figure 2: The same toy model wavefunctions after rescaling the size of the real line. The loci and breadths of the wavefunctions have also been rescaled and so the relative shapes are unaltered. However the physical Yukawa coupling scales parametrically with the size of the real line and its absolute magnitude has changed.

central values of the wavefunctions unaltered. The new wavefunctions are

$$\psi_a(x) = \frac{1}{\pi^{1/4} \lambda^{1/2} a^{1/2}} e^{-\frac{(x-\lambda\xi_a)^2}{2(\lambda a)^2}}, \quad (2.10)$$

$$\psi_b(x) = \frac{1}{\pi^{1/4} \lambda^{1/2} a^{1/2}} e^{-\frac{(x-\lambda\xi_b)^2}{2(\lambda a)^2}}, \quad (2.11)$$

$$\psi_c(x) = \frac{1}{\pi^{1/4} \lambda^{1/2} a^{1/2}} e^{-\frac{(x-\lambda\xi_c)^2}{2(\lambda a)^2}}. \quad (2.12)$$

The rescaled wavefunctions (for $\lambda = 2$) are illustrated in figure 2.

However, the physical Yukawa couplings do alter under this rescaling,

$$\hat{Y}_{abc} \rightarrow \frac{\hat{Y}_{abc}}{\sqrt{\lambda}}. \quad (2.13)$$

Note we can determine the scaling in (2.13) without ever having to evaluate the integral in (2.9). In a IIB context, computing the integral in (2.9) corresponds to computing the complete Yukawa coupling and would require a full-fledged Calabi-Yau computation. However the scaling of the physical Yukawas on the Kähler moduli can be much simpler, and as in (2.13) we can hope to compute it through elementary arguments.

In the framework of $\mathcal{N} = 1$ supergravity, we could now use the result of (2.13) to deduce the dependence of the matter metrics on the ‘Kähler moduli’. However we now seek to move beyond toy examples and apply the above strategy to IIB Calabi-Yau flux compactifications.

3. The large-volume model

We now apply the above ideas to realistic examples. We will use two geometries, first that of the large-volume models of [14, 15] and then that of the torus. As the first involves a

full Calabi-Yau geometry, there is no direct approach to computing bifundamental matter metrics.

3.1 Kähler metrics

We start this section with a brief description of the geometry of the large-volume models. These models exist within the framework of IIB flux compactifications with D3 and D7 branes. The Kähler potential and superpotential for the moduli take the standard form [10, 19, 17, 18],

$$\hat{K}(\Phi, \bar{\Phi}) = -2 \ln \left(\mathcal{V} + \frac{\hat{\xi}}{2g_s^{3/2}} \right) - \ln \left(i \int \Omega \wedge \bar{\Omega} \right) - \ln(S + \bar{S}). \quad (3.1)$$

$$\hat{W}(\Phi) = \int G_3 \wedge \Omega + \sum_i A_i e^{-a_i T_i}. \quad (3.2)$$

\mathcal{V} is the Einstein-frame volume of the Calabi-Yau. We use Φ to denote an arbitrary modulus field and do not specify the total number of moduli. The dilaton and complex structure moduli are stabilised by fluxes. The Kähler moduli are stabilised by a combination of α' corrections and nonperturbative superpotentials. As shown in [14, 15], these very generally interact to produce one exponentially large cycle controlling the overall volume together with $h^{1,1} - 1$ small cycles. We denote the large and small moduli by $T_b = \tau_b + ic_b$ and $T_i = \tau_i + ic_i$ respectively, with $i = 1 \dots h^{1,1} - 1$. Consistent with this, we assume the volume can be written as

$$\mathcal{V} = \tau_b^{3/2} - h(\tau_i), \quad (3.3)$$

where h is a homogeneous function of the τ_i of degree 3/2. The simplest model, whose properties have been studied in detail in [14, 15, 20], involves the manifold $\mathbb{P}_{[1,1,1,6,9]}^4$ and has $h^{1,1} = 2$ with

$$\mathcal{V} = \tau_b^{3/2} - \tau_s^{3/2}. \quad (3.4)$$

The large volume lowers both the string scale and gravitino mass,

$$m_s \sim \frac{M_P}{\sqrt{\mathcal{V}}} \quad \text{and} \quad m_{3/2} \sim m_{\text{soft}} \sim \frac{M_P}{\mathcal{V}}.$$

The stabilised volume is exponentially sensitive to the stabilised string coupling, $\mathcal{V} \sim e^{\frac{c}{g_s}}$, and may thus take arbitrary values. A volume $\mathcal{V} \sim 10^{15} l_s^6 \equiv 10^{15} (2\pi\sqrt{\alpha'})^6$ is required to explain the weak/Planck hierarchy and give a TeV-scale gravitino mass. As $m_s \gg m_{3/2}$, the low-energy phenomenology is that of the MSSM and thus the computation of matter metrics is an important part of the phenomenology.

We will not review the details of the moduli stabilisation here, leaving those to the references [14, 15]. Our interest here is rather in computing matter metrics and their dependence on the geometry. This geometry is illustrated in figure 3. The stabilised volumes of the small cycles are $\tau_s \sim \ln \mathcal{V}$. D7-branes wrapped on such cycles have gauge couplings qualitatively similar to those of the Standard Model. If the branes are magnetised, Standard Model chiral matter can arise from strings stretching between stacks of D7 branes.

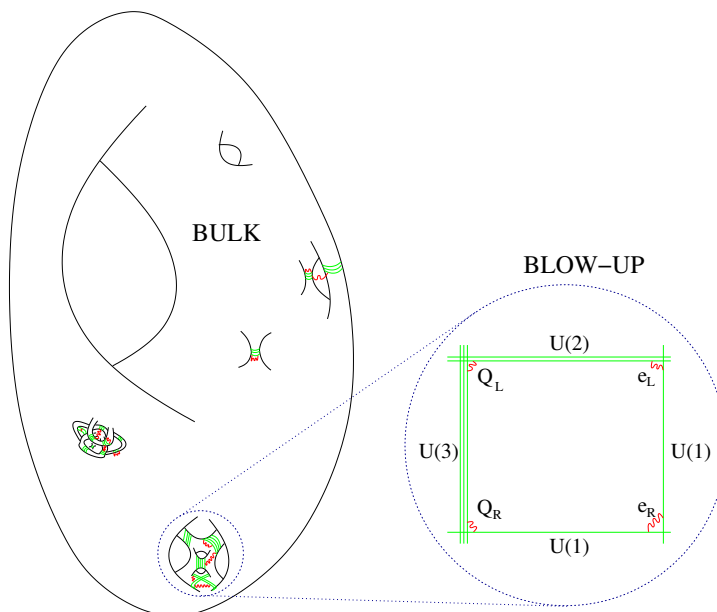


Figure 3: The physical picture: Standard Model matter is supported on a small blow-up cycle located within the bulk of a very large Calabi-Yau. The volume of the Calabi-Yau sets the gravitino mass and is responsible for the weak/Planck hierarchy.

We assume the Standard Model arises from a stack of magnetised branes wrapping one (or more) of the small cycles. Given this assumption, our aim is to compute the modular weights of the matter metrics for the bifundamental chiral matter.

In what we shall call the ‘minimal model’, there exists only one small blow-up 4-cycle on which a stack of magnetised D7 branes are wrapped. The existence of only one small cycle need not be incompatible with the several different gauge factors of the Standard Model. The spectrum of chiral fermions depends on the magnetised flux F present on the brane worldvolume. This is quantised on 2-cycles Σ_i ,

$$\int_{\Sigma_i} F \in \mathbb{Z}. \tag{3.5}$$

If several such 2-cycles exist within the 4-cycle, different brane stacks can be realised through different choices of 2-form flux on these 2-cycles. This is consistent with there being only one small cycle, as these 2-cycles may be homologically trivial within the Calabi-Yau and only non-trivial when restricted to the 4-cycle. As the cycle is a blow-up cycle, the branes cannot move off the cycle and have no adjoint matter. This permits a chiral spectrum, as found in explicit models of branes at singularities [21]. The geometry of this minimal model is shown in figure 4.⁴ We now address the computation of the modular weights.

⁴The minimal model is unlikely to be suitable for generating the exact MSSM gauge group. This is because a four-brane stack has four U(1) factors, three of which must become global and massive in order to give the single U(1) factor of the MSSM. U(1)s become global through the Green-Schwarz mechanism, but this requires the internal 2-cycles to be nontrivial within the Calabi-Yau. We thank the referee for drawing our attention to this point.

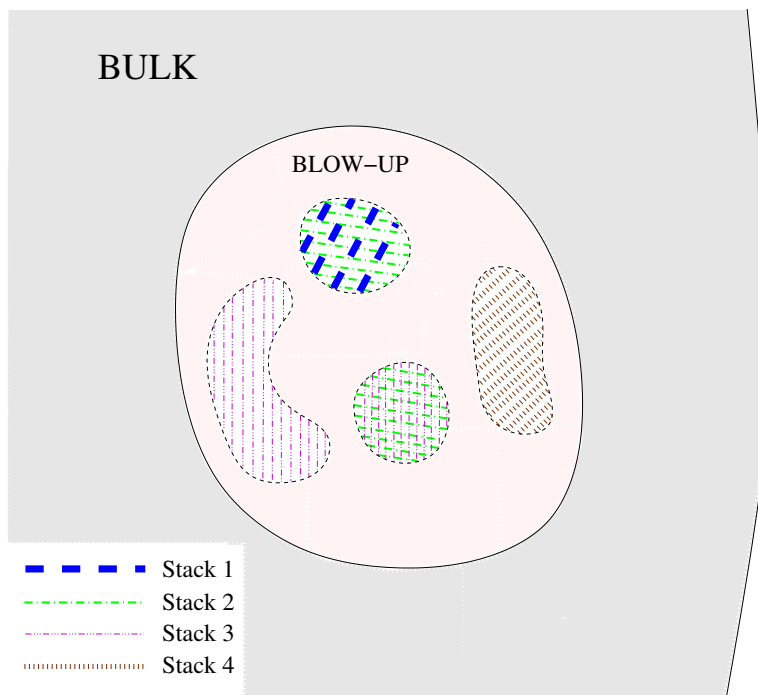


Figure 4: In the minimal geometry, there is only one small 4-cycle. The different brane stacks of the Standard Model are distinguished by having different magnetic fluxes on the internal 2-cycles of the 4-cycle. In the minimal model above, these 2-cycles are not inherited from the Calabi-Yau and only exist as cycles in the geometry of the 4-cycle. Four distinct brane stacks are required to realise the Standard Model, and we schematically show how these stacks are distinguished by different choices of magnetic flux.

3.1.1 Volume dependence

In the large-volume limit we can factorise \tilde{K}_α as in (2.5),

$$\tilde{K}_\alpha = \tau_b^{-p_\alpha} k_\alpha(\tau_i, \phi). \tag{3.6}$$

$\tau_b \sim \mathcal{V}^{2/3}$ is the size of the large 4-cycle and we use ϕ to denote both dilaton and complex structure moduli. While we have included an index p_α , from the picture of figure 3 we expect universality as locality implies all matter flavours should see the overall volume in the same way. We now argue that actually $p_\alpha = 1$.

To do so let us analyse the expressions for the physical Yukawa couplings. Using $\hat{K} = -2 \ln \mathcal{V}$ in (2.3), we obtain

$$\hat{Y}_{\alpha\beta\gamma} = \frac{x Y_{\alpha\beta\gamma}}{(k_\alpha k_\beta k_\gamma(\tau_i, \phi_i))^{1/2}} \tau_b^{\frac{-3+(p_\alpha+p_\beta+p_\gamma)}{2}}, \tag{3.7}$$

where x is $\mathcal{O}(1)$ and defined by $x\mathcal{V} = \tau_b^{3/2}(1 + \dots)$.

In the large-volume scenario illustrated in figures 3 and 4, the Standard Model branes are all supported around a localised (set of) small cycle(s) within a very large bulk. The physical origin of Yukawa couplings is through the interaction and overlap of the quantum

wavefunctions associated with the different matter fields. Matter fields supported on branes are localised on the branes, and thus the wavefunctions for Standard Model matter all have support in the local geometry on the small 4-cycle. As the interactions are determined only locally, in the large-volume limit the physical Yukawa couplings should be independent of the overall volume, provided that the local geometry is unaltered.

This argument is equivalent to saying that it is consistent to decouple gravity by taking $M_P/m_s \rightarrow \infty$ and study the field theory on the branes. The decoupling of gravity, which is achieved by taking the volume of the Calabi-Yau to infinity, does not force the physical Yukawa couplings to vanish. Such a situation is familiar from the study of branes at singularities, where the low-energy theory on the brane is well-defined and non-trivial even though the Calabi-Yau is non-compact.

The effect of the above is to tell us that the physical Yukawa couplings $\hat{Y}_{\alpha\beta\gamma}$ of (3.7) should be invariant under rescalings $\tau_b \rightarrow \lambda\tau_b$. This implies

$$p_\alpha + p_\beta + p_\gamma = 3$$

for all matter fields present in the Yukawa couplings. As noted earlier, to make this deduction it is crucial that the superpotential Yukawa couplings cannot depend on T . In the scenario of figures 3 and 4, all Standard Model matter arises as localised bifundamental D7-D7 states, and thus should experience the overall volume in the same way. We therefore expect p_α to be universal, giving

$$p_\alpha = 1 \quad \forall \alpha. \tag{3.8}$$

Equation (3.8) completely determines the modular weight of the matter fields with regard to the overall volume.

3.1.2 Small cycle dependence: the minimal model

We now address the calculation of the modular dependence on the small moduli for chiral bifundamental matter. We aim to compute the leading power-law dependence for the minimal model, working in the dilute flux approximation.

By performing a series expansion in τ_s , we can write

$$\tilde{K}_{\alpha\bar{\beta}} = \frac{\tau_s^\lambda}{\mathcal{V}^{2/3}} k_{\alpha\bar{\beta}}(\phi). \tag{3.9}$$

As in the dilute flux approximation the $\frac{1}{\tau_s}$ expansion is the perturbative weak coupling expansion, we know the expression (3.9) will be valid for large values of τ_s . Corrections to (3.9) subleading in τ_s will be suppressed at large cycle volume. The physical Yukawa couplings are given by (2.3)

$$\hat{Y}_{\alpha\beta\gamma} = e^{\hat{K}/2} \frac{Y_{\alpha\beta\gamma}}{(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma)^{\frac{1}{2}}}. \tag{3.10}$$

To obtain λ , it is therefore sufficient to obtain the scaling of $\hat{Y}_{\alpha\beta\gamma}$ with τ_s .

In the minimal model, we assume that the Standard Model comes from dimensional reduction of a stack of D7 branes wrapped on the small 4-cycle. The chiral spectrum

can in principle be found by dimensional reduction of the higher dimensional super Yang-Mills action in the presence of magnetic fluxes. This gives an explicit realisation of the understanding of Yukawas as due to the overlap of normalised wavefunctions, analogous to the computation of Yukawa couplings in either the heterotic string [22] or for D9 branes, for which this problem has been treated very explicitly in [23]. The action to be reduced is the DBI action, which in the dilute-flux approximation reduces to that of super Yang-Mills, whose fermionic terms include

$$\int_{\mathbb{M}_4 \times \Sigma} \bar{\lambda} \Gamma^i (\partial_i + A_i) \lambda. \tag{3.11}$$

We drop precise numerical factors of π or α' . The higher dimensional gauge field (A_i) and gaugino (λ) can be decomposed in a dimensional reduction,

$$A_m = \sum_i \phi_{4,i} \otimes \phi_{6,i} \quad \lambda = \sum_i \psi_{4,i} \otimes \psi_{6,i}. \tag{3.12}$$

We are most interested in the spectrum of massless chiral fermions in four dimensions. This is determined by counting the number of solutions of the Dirac equation on the cycle in the presence of magnetic fluxes. This is given by an index theorem and is topological, depending only on the cycle geometry and the magnetic fluxes. This specifies both the number and charge of the fermions present, and these quantities are invariant under continuous deformations of the cycle.

Direct reduction of the action (3.11) gives both the kinetic terms

$$\mathcal{L}_{\text{kin}} \sim \bar{\psi} \partial \psi \tag{3.13}$$

and the Yukawa couplings

$$\mathcal{L}_Y \sim \phi \bar{\psi} \psi. \tag{3.14}$$

The magnitude of the physical Yukawa couplings is determined by the relative magnitude of these two terms. Note that the physical Yukawas are dimensionless quantities and can be determined without any reference to the Planck scale or the normalisation of the gravitational action.

A full calculation of the Yukawa couplings requires the explicit scalar and fermion wavefunctions. We suppose we have solved the Dirac and Laplace equations,

$$\Gamma^i D_i \psi_A = \Gamma^i D_i \psi_B = \nabla^2 \phi_C = 0, \tag{3.15}$$

where D_i and ∇^2 are the appropriate differential operators on the fluxed 4-cycle. From (3.11), the kinetic term for the four-dimensional fermion ψ_A is

$$\left(\int_{\Sigma} \psi_{6,A}^\dagger \psi_{6,A} \right) \int_{\mathbb{M}_4} \bar{\psi}_{4,A} \Gamma^\mu \partial_\mu \psi_{4,A}. \tag{3.16}$$

Normalisation of the kinetic terms then requires that

$$\int_{\Sigma} \psi_{A,6}^\dagger \psi_{A,6} = \int_{\Sigma} \psi_{B,6}^\dagger \psi_{B,6} = \int_{\Sigma} \phi_6^* \phi_6 = 1. \tag{3.17}$$

The four-dimensional Yukawa couplings are also determined by the action (3.11),

$$\left(\int_{\Sigma} \bar{\psi}_A \Gamma^i A_{i,C} \psi_B \right) \int_{\mathbb{M}_4} \phi_C \bar{\psi}_A \psi_B. \quad (3.18)$$

The physical magnitude of the Yukawa coupling \hat{Y}_{ABC} are given by the overlap integral of normalised wavefunctions

$$\hat{Y}_{ABC} = \int_{\Sigma} \bar{\psi}_A \Gamma^i A_{i,C} \psi_B. \quad (3.19)$$

Our interest is the scaling of (3.19) with the cycle volume. Under a rescaling $\tau_s \rightarrow \beta \tau_s$, it follows from (3.17) that the normalised wavefunctions scale as

$$\psi_A \rightarrow \frac{\psi_A}{\sqrt{\beta}}. \quad (3.20)$$

The physical Yukawas then scale as

$$\hat{Y}'_{ABC} \sim \int_{\Sigma} (\beta d^4 y) \left(\frac{\psi_A}{\sqrt{\beta}} \right) \left(\frac{\psi_B}{\sqrt{\beta}} \right) \left(\frac{\phi_C}{\sqrt{\beta}} \right) = \frac{\hat{Y}_{ABC}}{\sqrt{\beta}}. \quad (3.21)$$

One may worry that under rescaling of the cycle volume the wavefunctions would undergo far more dramatic changes than the simple rescaling of (3.20). In the limit of dilute fluxes and large cycle volumes, this cannot occur. If the wavefunctions were to change their shape, rather than just their normalisation, the physical Yukawa couplings would also change far more dramatically than the simple scaling of (3.21). However, this cannot occur. The texture of the Yukawa couplings comes from the superpotential, and thus cannot depend on the Kähler moduli. They can only be changed by a change in the complex structure moduli, which has not occurred. The Kähler moduli can only affect the physical Yukawa couplings through the power λ of (3.9), which corresponds purely to an overall scaling of the wavefunctions and not to a change in the shape.

As the cycle size becomes smaller, quantum corrections due to the gauge group on the brane become important. These can alter the shape of the various wavefunctions - this corresponds to subleading powers of τ_s in (3.9). However, in the limit of large cycle volumes and dilute fluxes, this effect goes away and the wavefunctions become the purely classical ones with scaling behaviour given by (3.20).

The result (3.21) implies that the scaling of the physical Yukawas with the cycle volume is

$$\hat{Y}_{\alpha\beta\gamma} \sim \frac{\hat{Y}_{\alpha\beta\gamma}}{\sqrt{\tau_s}}. \quad (3.22)$$

This same dimensional reduction implies that the physical Yukawas do not scale with the overall volume. This is a calculational illustration of our earlier point that the Yukawa interaction is local and so should be insensitive to the bulk volume.

Comparison with equation (3.10) then shows that the matter metric must scale as

$$\tilde{K}_{\alpha}(\tau_s) \sim \frac{\tau_s^{1/3}}{\mathcal{V}^{2/3}} k_{\alpha}(\phi). \quad (3.23)$$

Here we note that nothing in our analysis has depended on whether the matter metric is diagonal or otherwise. The flavour structure is encoded in the superpotential and thus is only seen by the complex structure moduli. We can perform a rotation of the matter fields to diagonalise the kinetic terms, absorbing the non-diagonality in the (unknown) Yukawa couplings. Thus the scaling behaviour of (3.23) also applies to general non-diagonal metrics. For the minimal model, this therefore determines the matter metric in the large cycle volume dilute flux approximation to be of the form

$$\tilde{K}_{\alpha\bar{\beta}} = \frac{\tau_s^{1/3}}{\mathcal{V}^{2/3}} k_{\alpha\bar{\beta}}(\phi). \tag{3.24}$$

While the powers in (3.24) are in principle only the leading terms in a power series expansion, they dominate in a weak coupling expansion.

How large are the subleading terms? As in the dilute flux approximation τ_s controls the gauge coupling on the branes, we should interpret the series expansion in τ_s as an expansion in the coupling of the gauge theory. For a theory with gauge coupling

$$\alpha = \frac{g^2}{4\pi},$$

loop effects are suppressed by a factor $\frac{\alpha}{2\pi} \equiv \frac{g^2}{8\pi^2}$. For wrapped D7 branes, reduction of the DBI action gives

$$\alpha = \frac{1}{2\tau}, \tag{3.25}$$

and so loop corrections are suppressed by $\sim 4\pi\tau \sim 100$ and are at the percent level. Thus this suggests that the expansion in powers of τ is a well-controlled expansion. As the size of τ is determined by matching onto the observed gauge couplings, this is simply the statement that the Standard Model gauge couplings at $\Lambda \sim 10^{11}\text{GeV}$ (in the large-volume models, this is the string scale required for TeV-scale supersymmetry) lie deep in the perturbative regime.

Let us discuss the assumptions made in deriving (3.24). The first assumption was that of locality - the strength of the physical Yukawa interaction is insensitive to the overall volume. The justification for this is that all chiral matter is localised around the small cycle and thus the interactions are localised as well. This assumption completely determines the power of the volume that appears in (3.24). The second assumption was that of the minimal model - all chiral matter arises from dimensional reduction of a single stack of magnetised branes. This determined the power $\tau_s^{1/3}$ in (3.24). It seems difficult to escape the first assumption. However if the local geometry is more complicated than that of the minimal model, this second assumption may not hold. We now investigate some other possibilities for the local geometry and how they would alter the power λ .

3.1.3 Small cycle dependence: more complicated geometries

In the previous section, we assumed all D7s giving rise to the Standard Model wrapped an identical 4-cycle. If this does not hold, we would expect that (3.24) could be altered. We can envisage a situation in which the D7s are wrapping different small 4-cycles that are

however localised in a region of the CY, with volumes that are small and approximately equal. Under this assumption, one can still obtain approximate but concrete expressions similar to (3.24).

While we cannot now just reduce a single higher-dimensional action, we again expect that Yukawa couplings will arise from the overlap of normalised wavefunctions. These wavefunctions are supported on the pair-wise intersection locus of D7 branes, while the Yukawa coupling is supported on the triple intersection locus.

Three D7 branes intersecting at a point

Since one would not expect Yukawa couplings to arise in IIB string theory from non-intersecting branes⁵ the minimal scenario is three stacks of D7s (named for concreteness a , b and c), each wrapping a small 4-cycle in the Calabi-Yau, intersecting pairwise in 2-cycles (labeled ab , bc and ca) and whose triple intersection is a single point. The wavefunctions corresponding to the chiral fermions arising in the overlap of each pair of stacks have support only in the intersection 2-cycle, and hence their dependence on the 2-cycle volume must be

$$\psi_{ij}^\alpha(z_{ij}) \sim \frac{1}{\sqrt{A_{ij}}}, \tag{3.26}$$

with z_{ij} the complex coordinate parametrising the 2-cycle, $ij = ab, bc, ca$, and A_{ij} is the 2-cycle volume. α labels the family replication of the corresponding wave functions. The interactions of these are distinguished, as already emphasised, only by the complex structure moduli. Assuming that the triple intersection point is given by $(z_{ab}^0, z_{bc}^0, z_{ca}^0)$, the Yukawa coupling will just be the product of the wave functions evaluated at this point:

$$\hat{Y}_{\alpha\beta\gamma} = \psi_{ab}^\alpha(z_{ab}^0)\psi_{bc}^\beta(z_{bc}^0)\psi_{ca}^\gamma(z_{ca}^0) \sim \frac{1}{\sqrt{A_{ab}A_{bc}A_{ca}}} \sim \tau_s^{-3/4}, \tag{3.27}$$

where we have further assumed that all volumes are of similar size and are related to some characteristic 4-cycle volume τ_s . Assuming the same behaviour as in (3.9), we readily find $\lambda = 1/2$ and hence

$$\tilde{K}_\alpha \sim \frac{\tau_s^{1/2}}{\mathcal{V}^{2/3}} k(\phi). \tag{3.28}$$

The power of λ is increased compared to the case of the minimal model. As the branes wrap different cycles in this example, we would expect that ‘ τ_s ’ as appears in (3.28) should be expanded to be a function of the several moduli corresponding to the different cycles.

Another possibility is to have three stacks of branes whose common intersection is a 2-cycle. There are several possibilities here, some of them not easy to analyse, but there are two cases whose behaviour can be extracted straightforwardly. We follow the notation of the previous subsection.

⁵Contrary to the IIA case, in which Yukawa couplings can be generated among three D6-branes with no common intersection by world-sheet instanton amplitudes [24], these kind of contributions cannot appear in IIB orientifolds. The reason is that any world-sheet instanton contribution to the superpotential must be holomorphic in $\int_\Sigma (J + iB)$, Σ being the relevant (area minimising) 2-cycle wrapped by the world-sheet, J the Kahler form and B the B -field. But in IIB orientifold constructions the internal B -field is projected out and hence these contributions are absent.

Two branes overlapping on a 4-cycle

Suppose branes a and b overlap on a 4-cycle Π_s whose volume is given by the Kähler modulus τ_s , and brane c wraps a different 4-cycle Π_a whose intersection with Π_s is a 2-cycle whose area is denoted by A . The corresponding wave functions scale as

$$\begin{aligned} \psi_{ab} &\sim \frac{1}{\sqrt{\tau_s}}, \\ \psi_{bc} &\sim \frac{1}{\sqrt{A}}, \\ \psi_{ca} &\sim \frac{1}{\sqrt{A}}. \end{aligned} \tag{3.29}$$

Hence the Yukawa coupling scales as

$$\hat{Y}_{\alpha\beta\gamma} = \int_{\Pi_A} \psi_{ab}\psi_{bc}\psi_{ca} \sim \frac{1}{\sqrt{\tau_s}}. \tag{3.30}$$

From this result we again get $\lambda = 1/3$ and the dependence of the Kähler metric as

$$\tilde{K}_\alpha \sim \frac{\tau_s^{1/3}}{\mathcal{V}^{2/3}} k(\phi). \tag{3.31}$$

Three branes intersecting pairwise on the same 2-cycle

We now suppose we have three branes wrapping different cycles, such that the any pair of these branes intersect in the same 2-cycle Σ . The three stacks therefore also intersect in Σ . Clearly the three wave functions scale as

$$\psi_{ij} \sim \frac{1}{\sqrt{A}}, \tag{3.32}$$

where A is the area of Σ . We find

$$\hat{Y}_{\alpha\beta\gamma} = \int_{\Sigma} \psi_{ab}\psi_{bc}\psi_{ca} \sim \frac{1}{\sqrt{A}} \sim \tau_s^{-1/4}. \tag{3.33}$$

Here τ_s is the four-dimensional volume of a characteristic local 4-cycle of the construction, such that (roughly) $A \sim \sqrt{\tau_s}$. We obtain $\lambda = 1/6$ and

$$\tilde{K}_\alpha \sim \frac{\tau_s^{1/6}}{\mathcal{V}^{2/3}} k(\phi). \tag{3.34}$$

As above we expect that due to the several cycles wrapped τ_s should be expanded to be a function of the several moduli corresponding to the different cycles.

A bound on λ

In all the constructions analysed above we have found a value for λ between 0 and 1. One could ask whether there is a physical reason for having λ within these limits. In fact this does seem to be the case.

From the above analyses the physical Yukawa couplings scale as

$$\hat{Y} \sim \frac{V_{123}}{\sqrt{V_{12}V_{23}V_{31}}}. \tag{3.35}$$

V_{123} is the volume of the brane triple intersection, and V_{ij} the volume of the pairwise intersection between stack i and j . The $V_{ij(k)}$ are non-decreasing functions of the characteristic small 4-cycle volume τ_s (parametrised at first order by powers $V_{ij(k)} \sim \tau_s^\alpha$, with $\alpha = 1$ if the relevant intersections are 4-cycles, $\alpha = 1/2$ if they are 2-cycles, etc). Note that, for a given value of τ_s , $V_{ijk} \subset V_{ij}$, since V_{123} characterises the volume of the triple intersection. Then, if we parametrise the scaling of the Yukawa coupling as

$$\hat{Y} \sim \tau^{-\beta} \tag{3.36}$$

for some real β , we see that necessarily $\beta \geq 0$. Now, assuming a dependence of the Kähler metrics of the fields with τ_s like $\tilde{K} \sim \tau_s^\lambda$, we find $\lambda = 2\beta/3$, and hence $\lambda \geq 0$.

We can also extract an upper bound on λ by similar arguments. An upper bound on λ implies an upper bound on β . This will be attained whenever the numerator in (3.35) is minimised and the denominator maximised. Clearly the denominator is maximised whenever all $V_{ij} \sim \tau_s$, and the numerator will be minimised when $V_{123} \sim 1$. Irrespective of whether this can be realised or not, this is clearly the strongest dependence possible, since V_{123} cannot scale negatively with the volume. This dependence implies $\beta \leq 3/2$ and $\lambda \leq 1$. Hence we conclude that $\lambda \in [0, 1]$.⁶

3.2 Vanishing of the μ term

We now also argue, using similar scaling arguments as above, that in the large-volume models the superpotential μ term should vanish. Going from supergravity to field theory, the physical (i.e. normalised) μ parameter is given by

$$\hat{\mu} = \left(e^{\hat{K}/2} \mu + m_{3/2} Z - F^{\bar{m}} \partial_{\bar{m}} Z \right) (\tilde{K}_{H_1} \tilde{K}_{H_2})^{-\frac{1}{2}}, \tag{3.37}$$

where the F -terms are given by:

$$F^m = e^{\hat{K}/2} \hat{K}^{m\bar{n}} D_{\bar{n}} \tilde{W}. \tag{3.38}$$

We write

$$\tilde{K}_{H_1} = \tau_b^{-p_1} k_{H_1}(\tau_i), \tag{3.39}$$

$$\tilde{K}_{H_2} = \tau_b^{-p_2} k_{H_1}(\tau_i), \tag{3.40}$$

$$Z = \tau_b^{-p_z} z(\tau_i). \tag{3.41}$$

⁶The bound $\lambda < 1$ also follows from the requirement of a good classical limit, $\tau_b \rightarrow \infty, \tau_s \rightarrow \infty, \tau_b/\tau_s$ constant, in which the Kähler metric does not diverge.

We do not yet impose $p = 1$ because this is helpful in seeing the calculational structure. The physical μ term is then from (3.37) found to be

$$\hat{\mu} = \left(e^{K/2} \mu + m_{3/2} Z - F^{\bar{m}} \partial_{\bar{m}} Z \right) \left(\tilde{K}_{H_1} \tilde{K}_{H_2} \right)^{-\frac{1}{2}} \tag{3.42}$$

$$= \frac{\tau_b^{\frac{p_1+p_2}{2}}}{(k_{H_1} k_{H_2}(\tau_i))^{\frac{1}{2}}} \left(e^{\hat{K}/2} \mu + m_{3/2} Z - F^{\bar{m}} \partial_{\bar{m}} Z \right) \tag{3.43}$$

$$= \frac{x \tau_b^{\frac{p_1+p_2-3}{2}}}{(k_{H_1} k_{H_2})^{\frac{1}{2}}} \mu + \frac{z}{(k_{H_1} k_{H_2})^{\frac{1}{2}}} \tau_b^{\frac{p_1+p_2}{2} - p_z} m_{3/2} - (F^{\bar{m}} \partial_{\bar{m}} Z) \frac{\tau_b^{\frac{p_1+p_2}{2}}}{(k_{H_1} k_{H_2})^{\frac{1}{2}}}. \tag{3.44}$$

We evaluate

$$F^{\bar{m}} \partial_{\bar{m}} Z = p_z m_{3/2} \tau_b^{-p_z} z + \tau_b^{-p_z} F^{\bar{i}} \partial_{\bar{i}} z. \tag{3.45}$$

Therefore

$$\hat{\mu} = x \frac{\tau_b^{\frac{p_1+p_2-3}{2}}}{(k_{H_1} k_{H_2})^{\frac{1}{2}}} \mu + \frac{\tau_b^{\frac{p_1+p_2}{2} - p_z}}{(k_{H_1} k_{H_2})^{\frac{1}{2}}} \left[z(1 - p_z) m_{3/2} - F^{\bar{i}} \partial_{\bar{i}} z \right]. \tag{3.46}$$

Equation (3.8) now implies that the superpotential μ term must vanish. By using (3.8) in (3.46), we see that the volume scaling of the first term of (3.46) is

$$\mu' \sim \mathcal{V}^{-1/3} \mu + \dots$$

However, recall that the string scale behaves with volume as

$$m_s \sim \mathcal{V}^{-1/2} M_P.$$

Thus for any non-zero value of μ we can make the physical mass μ' arbitrarily greater than the string scale by taking the classical large-volume limit $\mathcal{V} \rightarrow \infty$. As such behaviour is unphysical, the only consistent case is $\mu = 0$. Of course, the vanishing of the superpotential μ term in (3.46) does not imply the vanishing of the physical μ term, which can be generated by a non-zero Z in the Giudice-Masiero mechanism [26].

4. The single Kähler modulus case

In this section we restrict to the simplest case of one Kähler modulus. In particular, this is the original realisation of the KKLT scenario. We consider this case separately for two reasons. First, as the simplest case it is more often used in the literature and therefore it is useful to have an explicit expression for the Kähler metric for it. Secondly, the large volume scenario usually requires more than one Kähler modulus and therefore the results of the previous section do not directly apply to this case. In particular we cannot just assume the configuration of figure 3.

We can see here that the arguments of section (3.1.2) can still be used for the minimal model in which the Standard Model comes from dimensional reduction of a stack of D7 branes wrapped on a single 4-cycle of size $\tau = \frac{T+T^*}{2}$. The Kähler potential then can be written as

$$K = -3 \log(T + T^*) + \tilde{K}(T, T^*) C^* C + \dots, \tag{4.1}$$

with $\tilde{K}(T, T^*) = (T + T^*)^{-p}$. We are left with the task of determining the power p .

Following section (3.1.2) we can see again that the physical Yukawa couplings scale like $(T + T^*)^{-1/2}$ from overlapping wavefunctions. Then, using (3.10), $e^{\tilde{K}/2} = (T + T^*)^{-3/2}$ and the fact that the original superpotential Yukawa couplings Y do not depend on T , we get $p = 2/3$.

Therefore the Kähler potential to leading order in the Kähler modulus expansion looks like:

$$K = -3 \log(T + T^*) + \frac{C^*C}{(T + T^*)^{2/3}} + \dots \tag{4.2}$$

Notice that this argument did not use the exponentially large volume. Furthermore, it can easily be seen that this power $2/3$ will also appear in the large volume scenario if the D7 branes wrap the exponentially large cycle instead of a ‘small’ cycle as was assumed in the previous section. The reason for this is that the volume is dominated by the large modulus τ_b with $\mathcal{V} \sim \tau_b^{3/2}$, and therefore the Kähler potential for the multi-moduli case looks similar to the one in (4.2). This is also consistent with substituting τ_s by τ_b , with $\lambda = 1/3$ and $\mathcal{V} \sim \tau_b^{3/2}$ in (3.23).

5. Toroidal examples

In this section we apply a similar approach to the case of toroidal compactifications. This differs from the large volume setup considered earlier, since here it is not possible to localise a small 4-cycle within a large bulk. However, we will see how one can still get the correct dependence on the Kähler moduli for the matter Kähler metrics from the type of scaling arguments used earlier. Our results can be compared with the explicit, complete expressions obtained in [12, 7].

Consider a factorisable T^6 with three Kähler moduli denoted by t_i . These are related to the areas of the 2-tori by $t_i \sim A_j A_k$. Consider a system of three magnetised D7 branes, each wrapping a different pair of tori and being point-like in third one⁷. Being magnetised branes, chiral fermions arise from the overlap of each pair of branes. For example in the case of $D7_1$ and $D7_2$ branes, this fermion has support on the third torus. The corresponding normalised wave functions (only defined on the overlap between the branes) are given by⁸

$$\begin{aligned} \psi_{12}(z_3) &\sim \frac{1}{\sqrt{A_3}}, \\ \psi_{23}(z_1) &\sim \frac{1}{\sqrt{A_1}}, \\ \psi_{31}(z_2) &\sim \frac{1}{\sqrt{A_2}}, \end{aligned} \tag{5.1}$$

⁷In standard notation, we call these a branes $D7_1, D7_2, D7_3$, where $D7_1$ is point-like in the first torus and wraps the second and third tori.

⁸In the notation of [23] these wave functions would have been defined multiplied by two ‘square roots of δ -functions’. These $\sqrt{\delta}$ functions allow the Yukawa couplings to be defined as an integral over the whole T^6 , rather than only over the overlap space. We prefer to remove these delta functions for clarity and for notational consistency with the rest of the paper.

where the z_i are the corresponding complex coordinate of each torus. For clarity, we have dropped the wavefunction dependence on complex structure moduli which differ between flavours, but actually this can be explicitly computed. This calculation was performed in [23]; the generic, complete form of the wave functions is given by

$$\psi_{ij}^\alpha(z) = \left(\frac{2\text{Im}\tau|M|}{A_k^2} \right)^{1/4} e^{i\pi M(z+\zeta)\frac{\text{Im}(z+\zeta)}{\text{Im}\tau}} \cdot \vartheta \left[\begin{matrix} \frac{\alpha}{M} \\ 0 \end{matrix} \right] (M(z+\zeta), M\tau). \quad (5.2)$$

In this expression, z stands for the complex coordinate in the k^{th} torus, τ is the complex structure of this k^{th} torus and ζ are complexified Wilson lines degrees of freedom, depending also on the complex structure. $M \in \mathbf{Z}$ is the relative magnetic flux in the k^{th} torus, and α labels the different matter fields in the same family. ϑ is given by the Jacobi theta-function

$$\vartheta \left[\begin{matrix} a \\ b \end{matrix} \right] (\nu, \tau) = \sum_{l \in \mathbf{Z}} e^{\pi i(a+l)^2 \tau} e^{2\pi i(a+l)(\nu+b)}. \quad (5.3)$$

These wave functions are solutions both of the Dirac and Laplace equations on the magnetised torus. The purpose of including the wavefunction (5.2) is to emphasise the contrast between the functional dependence on the Kähler moduli A_k and the complex structure moduli τ .

If we suppose, without loss of generality, that the intersection point between the three D7s is located at $z_i = 0$, then the physical Yukawa coupling is

$$\hat{Y}_{\alpha\beta\gamma} \sim \psi_{12}(0)\psi_{23}(0)\psi_{31}(0) \sim \frac{1}{\sqrt{A_1 A_2 A_3}} \sim \frac{1}{(t_1 t_2 t_3)^{1/4}}. \quad (5.4)$$

We see that the product of the three wave functions is always proportional to $(t_1 t_2 t_3)^{-1/4}$, and (comparing with the explicit wave functions on (5.2)), this is the only place where the Kahler moduli appear, as expected. Hence, whereas the Kahler moduli only give rise to an overall scale of masses, the complex structure moduli are responsible for the structure of eigenvalues that eventually gives rise to the flavour structure of the model. We must emphasise that this is the first term in a volume expansion for the A_i and subleading contributions are to be expected, corresponding to quantum corrections to the wave function.

Let us try and derive the Kähler moduli dependence of the matter metrics. The Kähler potential for a torus is

$$K = -\log(s + \bar{s}) - \log(t_1 t_2 t_3) - \log \prod_{i=1}^3 (U_i + \bar{U}_i). \quad (5.5)$$

Relating the matter metrics to the physical Yukawa coupling through (2.3), we obtain

$$\tilde{K}_{12} \tilde{K}_{23} \tilde{K}_{31} \sim \frac{1}{\sqrt{t_1 t_2 t_3}}. \quad (5.6)$$

This is consistent with the exact results [7, 12], which give

$$\tilde{K}_{12} \sim \frac{1}{\sqrt{t_3}}, \text{ etc.} \quad (5.7)$$

We may also compare with the case of three, differently magnetised, D9 branes wrapping a T^6 , a case analysed in full detail in [23]. In this case the chiral fermions have support over the entire T^6 , with the wavefunctions being given by

$$\psi_{12} \sim \frac{1}{\sqrt{V}},$$

$$\psi_{13} \sim \frac{1}{\sqrt{V}},$$
(5.8)

$$\psi_{23} \sim \frac{1}{\sqrt{V}}.$$
(5.9)

Again, in this case we can see from the explicit expressions in [23] how the wavefunctions have a trivial dependence on the Kähler moduli but an intricate (and flavour-sensitive) dependence on the complex structure moduli. A sample wave function has the generic form of a product of three functions like (5.2), one for each of the tori in the factorisation⁹ $T^6 = T^2 \times T^2 \times T^2$. The physical Yukawa couplings thus scale as

$$\hat{Y}_{\alpha\beta\gamma} \sim \int_V d^6y \psi_{ab}(y)\psi_{bc}(y)\psi_{ca}(y) \sim \frac{1}{\sqrt{V}},$$
(5.10)

and so the matter metrics behave as

$$\tilde{K}_{ab}\tilde{K}_{bc}\tilde{K}_{ca} \sim \frac{1}{\sqrt{t_1t_2t_3}}.$$
(5.11)

These results are consistent with those of [23].

In the toroidal case, the above techniques are not able to fully determine the matter metrics \tilde{K}_{ab} , instead only giving the product $\tilde{K}_{ab}\tilde{K}_{bc}\tilde{K}_{ca}$. This is a consequence of the fact that the D7 branes wrap several cycles and that for toroidal examples it is not possible to separate the Yukawa interaction and the overall volume in the same way as for the large-volume models. It would be interesting if these techniques could be developed to give the individual matter metrics for the toroidal case.

6. Discussion and conclusions

In this paper we have developed techniques to compute modular weights for bifundamental chiral matter in Calabi-Yau flux compactifications. These techniques have applications to the computation of soft terms in gravity-mediated supersymmetry breaking. For chiral matter arising from a single stack of (magnetised) D7-branes wrapping a small cycle in the large-volume models of [14], we obtain

$$\hat{K}_{\alpha\bar{\beta}} = \frac{\tau_s^{1/3}}{\mathcal{V}^{2/3}} k_{\alpha\bar{\beta}}(\phi) + \dots,$$
(6.1)

where ϕ denotes the complex structure moduli. (6.1) is the leading term in a series expansion in τ_s^{-1} and \mathcal{V}^{-1} .

⁹This is in the simplest case of a factorisable T^6 with no non-Abelian Wilson lines. As can be checked in [23], the general expression becomes much more complicated.

We can see that the bifundamental fields behave like the D3 brane fields and D7 Wilson lines, in the large volume limit, rather than the D7 brane positions. This is consistent with the fact that bifundamental D7 fields can be localised whereas the adjoint fields, describing the position of the D7 branes in the bulk manifold, cannot.

Notice that this behaviour is also different from the one calculated for toroidal orbifolds. Had we used the toroidal case as representative of Kähler potentials in the general case we would have been misguided. The reason for the difference is that toroidal compactifications do not provide good examples of large volume compactifications where the standard model can be localised on a D7 brane, independent of the overall volume. The assumptions we made in the large volume case do not hold for tori. However we were still able to use our techniques regarding the structure of Yukawa couplings for toroidal cases and get results consistent with the literature, once the peculiarities of toroidal compactifications were considered.

We have also managed to extend our techniques to give an independent derivation of the vanishing of the μ term in the superpotential. Therefore substantial information can be obtained for Calabi-Yau compactifications even though explicit string calculations are not viable. This illustrates the power of our techniques.

Let us discuss the limitations of the above techniques. First, the above method is restricted to modular weights for those moduli that do not appear in the superpotential Yukawa couplings. Such moduli are those with a shift symmetry, as these cannot appear perturbatively in the superpotential and thus in $Y_{\alpha\beta\gamma}$. The moduli Kähler potential is generally known and this allows the physical Yukawa couplings to be directly related to the matter Kähler metrics. If moduli appear in the superpotential, then it is not possible to separate the behaviour of the physical Yukawas into superpotential and Kähler potential terms.

However, we still need to know the scaling behaviour of the physical Yukawas. This gives a second restriction, that the physical Yukawa couplings arise from essentially classical physics through wavefunction overlap. It is this that allowed us to compute the scaling behaviour of $\hat{Y}_{\alpha\beta\gamma}$ in sections 3.1 and 5. If the Yukawas were nonlocal effects arising from nonperturbative instanton effects (as does occur for IIA braneworlds), then it is not obvious how to compute the scaling of $\hat{Y}_{\alpha\beta\gamma}$.

Finally, the techniques all apply only to the classical weak-coupling limit. This is equivalent to determining the leading power λ of τ_s in the expansion (3.9). In IIB compactifications, τ_s controls the gauge coupling on D7 branes and so we expect the full expression of (3.9) to be a series expansion in τ_s . However if τ_s ceases to be large, then the knowledge of simply the leading power λ is inadequate as the expansion is not well controlled.

We can foresee many applications of our results given the fact that bifundamental fields are chiral and are expected to provide the physical observable particles in realistic models. One such application is to determine the structure of soft terms. One of the principal difficulties in computing soft terms in the large-volume models of [14] was the lack of knowledge of the matter metrics for bifundamental fields. This required the use of generic expressions in [15, 25, 20], referring to (for example) adjoint matter on D7 branes rather

than the bifundamental matter most relevant for the problem of MSSM soft terms. This has been addressed in section 3.1 of this paper and the results have obvious applications to the computation of soft terms that will be presented in a companion paper [27].

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